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CRACK INITIATION IN A STIFFENED PLATE

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The fracture mechanics problem of crack initiation in a stiffened plate is considered. The crack nucleus is modeled by a prefracture zone with bonds between the crack faces, which is treated as a region of weakened interparticle bonds of the material. The boundary-value problem of the equilibrium of a stiffened plate with a crack nucleus reduces to a nonlinear singular integrodifferential equation with a Cauchy type kernel. The strains in the crack initiation zone are found by solving this equation. The case of the stress state of the plate with a periodic system of prefracture zones is considered.

Key words: *prefracture zone, adhesion forces, strains in interfacial bonds, crack initiation.*

Sheet structures are usually fabricated from thin plates reinforced by riveted stiffening ribs to ensure sufficient strength. The deformation of an unbounded plate reinforced by a regular system of ribs of narrow rectangular cross sections has been the subject of extensive research (see, for example, [1–4]). Considerable attention has been given to the analysis of the fracture of a plate stiffened by a regular system of stringers [5–9]. In all indicated papers, the consideration was confined to a Griffith crack. At the same time, investigation of the initiation of a crack-type defect is of great significance.

Formulation of the Problem. We consider an elastic isotropic thin plate with transverse stiffening ribs (stringers) riveted at the points $z = \pm(2m+1)L \pm iny_0$ ($m = 0, 1, 2, \ldots$; $n = 1, 2, \ldots$). At infinity, the plate is subjected to homogeneous extension along the stringers by a stress $\sigma_y^\infty = \sigma_0$. For the stringers, we adopt the hypothesis of a one-dimensional continuum, which assumes that the stringer thickness does not vary under deformation and that the stress state of the stringer is uniaxial. The stringers do not resist bending and work only in tension.

The following assumptions are adopted:

1) In the thin-walled sheet structural member (plate), a plane stress state occurs;

2) The stiffening system of stringers is a truss, and the weakening of the stringers due to the attachment is ignored;

3) The sheet member and the stiffeners interact with each other in the same plane and only in the attachment zones;

4) All attachment zones (points) are identical, and their radius (the adhesion area) is small compared to the spacing between them and other characteristic sizes;

5) The effect of an attachment point on both the stringer and the plate is modeled by a point force.

In the computational scheme, the action of the stringers is modeled by unknown equivalent point forces applied at the points of connection of the ribs with the medium. As the stiffened plate is loaded, prefracture zones arise in the plate, which are modeled by regions of weakened bonds of the material. It is assumed that the prefracture zone is oriented toward the maximum tensile stress arising in the stiffened plate. The interaction of the faces of the prefracture zone is modeled by introducing bonds with a specified strain diagram between the faces of the prefracture zone. The physical nature of such bonds and the dimensions of the prefracture zone depend on the chosen material. Generally, the deformation law for the bonds is nonlinear [10–12].

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The problem of the stress–strain state of a deformed solid in which there are layers of overstrained material can be reduced to the problem of a stress–strain state in an elastic body weakened by a notch whose faces interact according to some law. With such an approach to solving the crack initiation problem, it is first of all necessary to establish the relation between the strains and displacements in the deformed material region in which there are forces of interparticle interaction (attraction between the faces). Generally, the solution of the problem is rather complicated.

In the case studied here, the initiation of a crack-type defect involves the transition of the prefracture zone to the region of broken bonds between the surfaces of the material. Since the prefracture zone is small compared to the remaining part of the stiffened plate, it can mentally be removed and replaced by a notch whose faces interact according to some law that corresponds to the action of the removed material. In this case, the dimensions of the prefracture zone are unknown and should be determined during the solution of the problem.

Studies of the regions with disturbed material structure have shown that in the initial stage, prefracture zones are a narrow elongated layer and an increase in the load results in a secondary system of zones containing material with partially broken bonds [13].

We consider a prefracture zone of length $2l$ located on an abscissa segment $y = 0$, $|x| \leqslant l$. The faces of the prefracture zone interact through the bonds and thus restrain the defect (crack) initiation. In the mathematical modeling of the interaction of the faces of the prefracture zone, we assume that between the faces there are bonds (adhesion forces), whose deformation law is specified. The action of external loads on the plate leads to the occurrence of strain $q(x)$ in the interfacial bonds in the prefracture zone. Because the problem is symmetric about the abscissa, these strains have only a normal component. The value of these stresses $q(x)$ and the dimension l of the prefracture zones are not known beforehand and are to be determined during the solution of the boundary-value problem of fracture mechanics.

In the problem considered, the edge conditions on the faces of the prefracture zone are given by

$$y = 0,\ |x| \leqslant l: \qquad \sigma_y - i\tau_{xy} = q(x). \tag{1}$$

The constitutive relations of the problem should be supplemented by an equation relating the opening of the faces of the prefracture zone and the strains in the bonds. Without loss of generality, this equation can be written as [14]

$$v^+(x,0) - v^-(x,0) = C(x,q)q(x), \tag{2}$$

where $v^+ - v^-$ is the normal component of the openings of the faces of the prefracture zone, x is the affix of the points of the prefracture zone, and $C(x,q)$ is the effective compliance of the bonds, which depends on their tension.

With the use of the Kolosov–Muskhelishvili formulas [15] and the edge conditions on the faces of the prefracture zone, the problem is reduced to determining two analytical functions $\Phi(z)$ and $\Psi(z)$ from the boundary conditions

$$y = 0,\ |x| \leqslant l: \qquad \Phi(x) + \overline{\Phi(x)} + \bar{x}\Phi'(x) + \Psi(x) = q(x). \tag{3}$$

To determine the values of the external load for which crack Initiation occurs, we need to supplement the formulation by the condition (criterion) of crack formation (rupture of the interparticle bonds of the material). As such a condition we use the criterion of the critical opening of the faces of the prefracture zone

$$v^+ - v^- = \delta_c, \tag{4}$$

where δ_c is the characteristic of the material resistance to crack formation. This condition allows us to determine the parameters of the stiffened plate for which a crack forms in the plate.

Solution of the Boundary-Value Problem. We seek a solution of the boundary-value problem (3) in the form

$$\varphi(z) = \varphi_0(z) + \varphi_1(z), \qquad \psi(z) = \psi_0(z) + \psi_1(z), \tag{5}$$

where $\Phi(z) = \varphi'(z)$; $\Psi(z) = \psi'(z)$; the functions $\varphi_0(z)$ and $\psi_0(z)$ define the stress and strain fields in the solid plate without a prefracture zone.

In the case considered, as $\varphi_0(z)$ and $\psi_0(z)$ we use the functions

$$\varphi_0(z) = \frac{1}{4}\,\sigma_0 z - \frac{i}{2\pi(1+\varkappa_0)h} \sum_{m=-\infty}^{\infty}{}' \sum_{n=-\infty}^{\infty}{}' P_{mn} \ln \frac{z - m_* L + iy_0 n}{z - m_* L - iy_0 n},$$

$$\psi_0(z) = \frac{1}{2}\,\sigma_0 z - \frac{i\varkappa_0}{2\pi(1+\varkappa_0)h} \sum_{m=-\infty}^{\infty}{}' \sum_{n=-\infty}^{\infty}{}' P_{mn} \ln \frac{z - m_* L + iy_0 n}{z - m_* L - iy_0 n} \tag{6}$$

$$- \frac{i}{2\pi(1+\varkappa_0)h} \sum_{m=-\infty}^{\infty}{}' \sum_{n=-\infty}^{\infty}{}' P_{mn} \Big(\frac{m_* L - iy_0 n}{z - m_* L - iy_0 n} - \frac{m_* L + iy_0 n}{z - m_* L + iy_0 n} \Big).$$

Here h is the plate thickness, y_0 is the spacing between the attachment points, L is half the distance between the stringers, $\varkappa_0$ is the elastic Muskhelishvili constant, $m_* = 2m+1$, and P_{mn} are the point forces to be determined; the prime at the summation symbol indicates that the index $n = m = 0$ is eliminated from the summation,.

To determine the analytical functions $\Phi_1(z)$ and $\Omega_1(z) = z\Phi_1'(z) + \Psi_1(z)$ based on (3), (5), and (6), we obtain the boundary-value problem

$$y = 0,\ |x| \leqslant l: \qquad \Phi_1(z) + \overline{\Phi_1(z)} + \Omega_1(z) = q(x) + f(x), \tag{7}$$

where $f(x) = -[\Phi_0(x) + \overline{\Phi_0(x)} + x\Phi_0'(x) + \Psi_0(x)]$.

Since the stresses in the elastic plate are bounded, the solution of the boundary-value problem (7) should be sought in the class of everywhere bounded functions. We note that by virtue of the symmetry of the problem about the axis Ox, the function $f(x)$ is real, and, hence, using (7) over the entire real axis, we have $\operatorname{Im}\Omega_1(z) = 0$. Therefore, taking into account the conditions at infinity, we obtain $\Omega_1(z) = 0$.

Thus, for the function $\Phi_1(z)$, we obtain the Dirichlet problem

$$\begin{aligned} y = 0,\ |x| \leqslant l: &\qquad \operatorname{Re}\Phi_1(z) = [f(x) + q(x)]/2, \\ z \to \infty: &\qquad \Phi_1(z) \to 0. \end{aligned} \tag{8}$$

The desired solution of problem (8) is written as

$$\Phi_1(z) = \frac{\sqrt{z^2 - l^2}}{2\pi i} \int_{-l}^{l} \frac{[f(x) + q(x)]\,dx}{\sqrt{x^2 - l^2}\,(x - z)}. \tag{9}$$

In view of the behavior of the function $\Phi_1(z)$ at infinity, the solvability condition for the boundary-value problem (8) has the form

$$\int_{-l}^{l} \frac{f(x)\,dx}{\sqrt{l^2 - x^2}} + \int_{-l}^{l} \frac{q(x)\,dx}{\sqrt{l^2 - x^2}} = 0. \tag{10}$$

This relation can be used to determine the dimension l of the prefracture zone.

Using formulas (6), we write the function $f(x)$ in explicit form

$$f(x) = -\sigma_0 + \frac{1}{\pi h} \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} P_{mn} \frac{n y_0}{n^2 y_0^2 + (x - m_* L)^2} \Big(\frac{3+v}{2} - (1+v)\, \frac{(x - m_* L)^2}{n^2 y_0^2 + (x - m_* L)^2} \Big)$$

$$+ \frac{1}{\pi h} \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} P_{mn}\, \frac{n y_0}{n^2 y_0^2 + (x + m_* L)^2} \Big(\frac{3+v}{2} - (1+v)\, \frac{(x + m_* L)^2}{n^2 y_0^2 + (x + m_* L)^2} \Big).$$

To determine the values of the point forces P_{mn} ($m = 1, 2, \ldots;\ n = 1, 2, \ldots$), we use Hooke's law, according to which the value of the point force P_{mn} exerted on each attachment point by the stiffening rib is equal to

$$P_{mn} = \frac{E_s F}{2 y_0 n}\, \Delta v_{mn}. \tag{11}$$

Here E_s is Young's modulus of the material of the stiffening rib, F is the cross-sectional area of the stiffening rib, $2y_0 n$ is the distance between the attachment points, and Δv_{mn} is the relative displacement of the attachment points, which is equal to the elongation of the corresponding segment of the stiffening rib.

We denote the radius of the attachment zone (point) by a and adopt the natural assumption that in the examined elastic problem, the relative displacement of the points $z = m_* L + i(n y_0 - a)$ and $z = m_* L - i(n y_0 - a)$ is equal to the relative displacement of the attachment points Δv_{mn} [16]. This additional condition of displacement compatibility allows one to find the solution of the problem formulated above.

Using the complex potentials (5), (6), and (9) and the Kolosov–Muskhelishvili formulas [15], we obtain the relative displacement of the attachment points Δv_{mn}:

$$\Delta v_{kr} = \Delta v_{kr}^0 + \Delta v_{kr}^1,$$

$$\Delta v_{kr}^0 = \frac{1}{2\pi(1+\varkappa_0)\mu h} \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} P_{mn} \Big[\varkappa_0 \ln \frac{C_1^2 C_2^2}{C_3 C_4 C_5 C_6}$$

$$+ b_2 b \Big(\frac{2k(k-m)L^2 + ab}{C_3 C_1} + \frac{2k(k+m)L^2 + ab}{C_4 C_2} \Big)$$

$$+ b_3 b_1 \Big(\frac{2k(k-m)L^2 + ab_1}{C_5 C_1} + \frac{2k(k+m)L^2 + ab_1}{C_6 C_2} \Big) \Big] + \frac{\sigma_0}{\mu} (1+\varkappa_0) b_4,$$

$$\Delta v_{kr}^1 = \frac{1}{\pi\mu} \int_0^l \frac{[f(t) + q(t)]F(t)}{\sqrt{l^2 - t^2}}\, dt.$$

Here

$$b = (r-n)y_0 - a, \quad b_1 = (r+n)y_0 - a, \quad b_2 = 2(r-n)y_0, \quad b_3 = 2(r+n)y_0,$$

$$b_4 = r y_0 - a, \quad C_1 = (k-m)^2 L^2 + a^2, \quad C_2 = (k+m)^2 L^2 + a^2, \quad C_3 = (k-m)^2 L^2 + b^2,$$

$$C_4 = (k+m)^2 L^2 + b^2, \quad C_5 = (k-m)^2 L^2 + b_1^2, \quad C_6 = (k+m)^2 L^2 + b_1^2,$$

$$F(t) = (1+\varkappa_0) f_1(t) + 2 b_4 f_2(t),$$

$$f_1(t) = D \sin\varphi + \sqrt{l^2 - t^2} \ln \frac{D^2 \cos^2\varphi + (D\sin\varphi - \sqrt{l^2 - t^2})^2}{D^2 \cos^2\varphi + (D\sin\varphi + \sqrt{l^2 - t^2})^2},$$

$$f_2(t) = \frac{D}{d^2 + d_1^2} [kL(d\cos\varphi - d_1 \sin\varphi) - b_4(d_1 \cos\varphi + d\sin\varphi)], \quad \varphi = \frac{1}{2} \arctan \frac{d_1}{B_1},$$

$$B_1 = k^2 L^2 - l^2 - b_4^2, \quad d_1 = 2kLb_4, \quad d = t^2 - k^2 L^2 + b_4^2, \quad D = \sqrt{A_1}\,, \quad A_1 = \sqrt{B_1^2 + d_1^2}\,.$$

The right side of relations (11) contains the unknown parameter l, which characterizes the length of the prefracture zone, and the strains $q(x)$ in the interfacial bonds in the prefracture zone.

To determine the complex potential $\Phi_1(z)$, it is necessary to find the strains $q(x)$ in bonds. Using the Kolosov–Muskhelishvili relation and the boundary value of the function $\Phi_1(z)$, on the segment $|x| \leqslant l$, we obtain the equality

$$\Phi_1^+(x) - \Phi_1^-(x) = \frac{2\mu i}{1+\varkappa_0} \frac{\partial}{\partial x} (v^+ - v^-). \tag{12}$$

Using the Sokhotsky–Plemelj formulas [15] and formula (9), we obtain

$$\Phi_1^+(x) - \Phi_1^-(x) = -\frac{i\sqrt{l^2 - x^2}}{\pi} \Big(\int_{-l}^{l} \frac{f(t) + q(t)}{\sqrt{l^2 - t^2}\,(t-x)}\, dt \Big). \tag{13}$$

Substituting expression (13) into the left side of Eq. (12), in view of relation (2), and performing some transformations, we obtain the following nonlinear integrodifferential equation for the unknown function $q(x)$:

$$-\frac{\sqrt{l^2 - x^2}}{\pi} \Big(\int_{-l}^{l} \frac{q(t)\, dt}{\sqrt{l^2 - t^2}\,(t-x)} + \int_{-l}^{l} \frac{f(t)\, dt}{\sqrt{l^2 - t^2}\,(t-x)} \Big) = \frac{2\mu}{1+\varkappa_0} \frac{\partial}{\partial x} (C(x,q) q(x)). \tag{14}$$

Numerical Algorithm and Analysis of the Solution. Equation (14) is a nonlinear integrodifferential equation with a Cauchy kernel and can be solved only numerically. Its solution can be made using a collocation scheme with approximation of the unknown function. In the case where the deformation law of the bonds is nonlinear, it is reasonable to determine the strains $q(x)$ [see (2)] in the bonds using an iterative scheme similar to that used in the method of elastic solutions [17].

To avoid solving the integrodifferential equation, we represent (14) as

$$-\frac{1+\varkappa_0}{2\mu}\int\limits_{-l}^{x} Q(x)\,dx = C(x,q)q(x), \tag{15}$$

where

$$Q(x) = \frac{\sqrt{l^2-x^2}}{\pi}\Big(\int\limits_{-l}^{l}\frac{q(t)\,dt}{\sqrt{l^2-t^2}\,(t-x)} + \int\limits_{-l}^{l}\frac{f(t)\,dt}{\sqrt{l^2-t^2}\,(t-x)}\Big).$$

We partition the segment $(-l,l)$ into M nodes t_m $(m=1,2,\ldots,M)$ and require that condition (15) be satisfied at these points. As a result, we obtain an algebraic system of M equations for the approximate values of $q(t_m)$ $(m=1,2,\ldots,M)$:

$$\begin{gathered} C_0 Q(t_1) = C(t_1, q(t_1))q(t_1), \\ C_0(Q(t_1)+Q(t_2)) = C(t_2, q(t_2))q(t_2), \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ C_0\sum_{m=1}^{M} Q(t_m) = C(t_M, q(t_M))q(t_M). \end{gathered} \tag{16}$$

Here

$$C_0 = -\frac{1+\varkappa_0}{2\mu}\,\frac{\pi l}{M}.$$

In the derivation of the algebraic system, the integration interval was reduced to the interval $[-1,1]$ and the integrals were then replaced by finite sums by means of the Gauss quadrature formulas.

Since the length l of the prefracture zones was unknown, the algebraic system (16) turned out to be nonlinear even for linearly elastic bonds. The obtained algebraic systems (11) and (16) and Eq. (10) are coupled and should be solved simultaneously. The integrals in Eqs. (11) and (10), were replaced by sums using the Gauss quadrature formula. In the case of linearly elastic bonds, their compliance $C(x,q)$ is constant. The algebraic system (11), (16) and (10) was solved using the method of successive approximations, which consists of the following. System (11), (16) is solved for a certain definite value l_* with respect to $M+N_1\times N_2$ unknowns $q_1^0, q_2^0, \ldots, q_M^0$ and $P_{11}, P_{12}, \ldots, P_{N_1N_2}$. The value of l_* and the obtained quantities $q_1^0, q_2^0, \ldots, q_M^0$ and $P_{11}, P_{12}, \ldots, P_{N_1N_2}$ are substituted into Eq. (10), which was earlier represented as a sum. An arbitrary value of the parameter l_* and its corresponding values of $q_1^0, q_2^0, \ldots, q_M^0$ and $P_{11}, P_{12}, \ldots, P_{N_1N_2}$, generally speaking, do not satisfy Eq. (10) of the system. Therefore, the calculations are repeated until they yield the value of the parameter l_* for which the last equation (10) of the system is satisfied with the specified accuracy.

In the case of a nonlinear deformation law for the bonds, the stains in the prefracture zone were determined using a method similar to the method of elastic solutions [17].

It is assumed that for $v^+ - v^- \leqslant v_*$, the deformation law of the interparticle bonds (adhesion forces) is linear.

The first step of the iterative calculation process consists of solving system (11), (16), (10) for linearly elastic interparticle bonds. The subsequent iterations are satisfied only if the inequality $v^+ - v^- > v_*$ holds on a certain part of the prefracture zone. For such iterations, the system of equations is solved in each step for quasielastic bonds with an effective compliance variable along the prefracture zone and dependent on the stains in the bonds obtained in the previous calculation step. The effective compliance is calculated in the same way as the secant modulus in the method of variable elastic parameters [18].

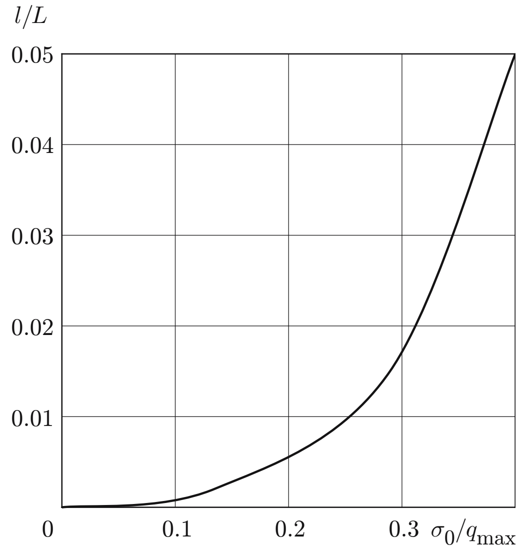

Fig. 1. Length of the prefracture zone l/L versus external loading $\sigma_0/q_{\max}$.

The iterative process is terminated when the strains along the prefracture zone obtained in two successive iterations become little different from each other.

The nonlinear part of the strain curve was approximated by a bilinear relation [14] whose ascending segment corresponded to the elastic deformation of the bonds ($0 < v^+ - v^- \leqslant v_*$) with their maximum tension. For $v^+ - v^- > v_*$, the deformation law was described by a nonlinear dependence determined by the points (v_*, σ_*) and (δ_c, σ_c); at $\sigma_c \geqslant \sigma_*$, this dependence became increasing linear and corresponding to the linear strengthening for the elastoplastic deformation of the bonds.

Figure 1 shows a curve of the length of the prefracture zone $d = l/L$ versus the dimensionless external loading $\sigma_0/q_{\max}$ for the following values of the free parameters of the problem: $\varepsilon_1 = a_0/L = 0.01$, $\varepsilon = y_0/L = 0.25$, $\nu = 0.3$, $E = 7.1 \cdot 10^4$ MPa (B95 alloy), $E_s = 11.5 \cdot 10^4$ MPa (60/40 aluminum/steel composite), $N_1 = N_2 = 14$, $M = 30$, $F/(y_0 h) = 1$, $v_* = 10^{-6}$ m, $\sigma_* = 130$ MPa, $\sigma_{\rm cr}/\sigma_* = 2$, and $\delta_c = 2 \cdot 10^{-6}$ m; effective compliance of the bonds $C = 2 \cdot 10^{-7}$ m/MPa.

Figure 2 shows the strain distributions in the bonds of the prefracture zone.

To determine the limiting state in which a crack forms, we use criterion (4). In this case, the condition determining the limiting value of the external load has the form

$$C(x_0, q)q(x_0) = \delta_c. \tag{17}$$

In the problem considered, one might expect that $x_0 = 0$, i.e., the point $x = x_0$ is at the center of the prefracture zone.

Simultaneous solution of the algebraic systems (11), (16), (10), and (17) provides (for specified fracture resistance characteristics of the material) the critical external load and the length of the prefracture zone l_c for the limit equilibrium state at which a crack appears.

Figure 3 gives a curve of the critical load $\sigma_0^*/\sigma_{\rm t}$ versus relative opening δ_*/l $[\delta_* = \pi\delta_c\mu/(1+\varkappa_0)\sigma_{\rm t}]$ in the center of the prefracture zone.

Periodic System of Prefracture Zones. Let in the stiffened plate under loading there is a periodic system of rectilinear prefracture zones of length $2l$ with a period ω, located on the abscissa. In the case studied, the initiation of crack nuclei is the process of transition of the prefracture zone to the region of broken bonds between the plate surfaces. The dimensions of the prefracture zones are not known beforehand and are to be determined during the solution of the problem.

The edge conditions on the faces of the periodic system of prefracture zones are written as

$$y = 0,\ |x - m\omega| \leqslant l: \qquad \sigma_y - i\tau_{xy} = q(x).$$

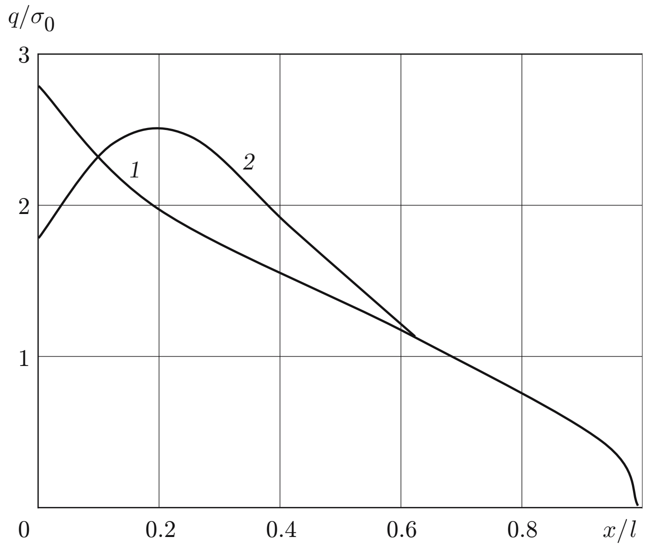

Fig. 2

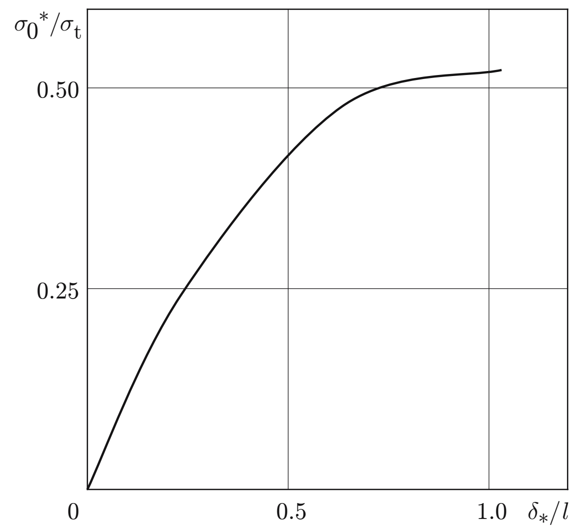

Fig. 3

Fig. 2. Distribution of the normal forces q/σ_0 in the interfacial bonds in the prefracture zone: 1) linear dependence; 2) bilinear dependence.

Fig. 3. Critical load $\sigma_0^*/\sigma_{\rm t}$ versus relative opening δ_*/l at the center of the prefracture zone.

In this case, a problem similar to the problem for one prefracture zone is solved. For the function $\Phi_1(z)$, we obtain the Dirichlet problem

$$\begin{aligned} y = 0,\ |x - m\omega| \leqslant l: &\qquad \operatorname{Re} \Phi_1(z) = [f(x) + q(x)]/2, \\ z \to \infty: &\qquad \Phi_1(z) \to 0. \end{aligned} \tag{18}$$

By means of the transformation $w = \sin(\pi z/\omega)$, we convert from the physical plane z to the parametric plane of the complex variable w. In this case, the appearance of the periodic system of prefracture zones of the plane z becomes an infinite-sheeted Riemann surface with a notch $(-l_0, l_0)$, where $l_0 = \sin(\pi l/\omega)$.

The required solution of problem (18) in the class of everywhere bounded functions is written as

$$\Phi_1(z) = \frac{X(z)}{2\pi i} \int\limits_{-l_0}^{l_0} \frac{f(x) + q(x)}{X(x)(\sin(\pi x/\omega) - \sin(\pi z/\omega))} \, \frac{\pi}{\omega} \, \cos \frac{\pi x}{\omega} \, dx.$$

Here $X(z)$ is a branch of the function $\sqrt{\sin^2(\pi z/\omega) - \sin^2(\pi l/\omega)}$ that has the form $\sin(\pi z/\omega)$ as $|z| \to \infty$.

In view of the behavior of the function $\Phi_1(z)$ at infinity, the condition of solvability of the boundary-value problem (18) is written as

$$\int\limits_{-l_0}^{l_0} \frac{f(x) + q(x)}{X(x)} \, \frac{\pi}{\omega} \, \cos \frac{\pi x}{\omega} \, dx = 0.$$

This condition is used to determine the length l of the prefracture zone.

In the case considered, we obtain the following nonlinear integrodifferential equation for the unknown function $q(x)$:

$$-\frac{X_*(x)}{\pi} \int\limits_{-l_0}^{l_0} \frac{f(t) + q(t)}{X_*(t)(\sin(\pi t/\omega) - \sin(\pi x/\omega))} \, \frac{\pi}{\omega} \, \cos \frac{\pi t}{\omega} \, dt = \frac{2\mu}{1 + \varkappa_0} \, \frac{\partial}{\partial x} \, [C(x, q) q(x)]. \tag{19}$$

Here $X_*(x) = \sqrt{\sin^2(\pi l/\omega) - \sin^2(\pi x/\omega)}$.

The sought values of the point forces are determined by solving the infinite system of equations (11). By virtue of the periodicity of the problem, this system degenerates into one infinite algebraic system for P_{m1} $(m = 1, 2, \dots)$. This leads to a change in the expressions for Δv_{mn}, which are cumbersome and not given here.

The nonlinear integrodifferential equation (19) are represented as

$$-\frac{1+\varkappa_0}{2\mu}\int\limits_{-l_0}^{x} Q_1(x)\,dx = C(x,q)q(x), \tag{20}$$

where

$$Q_1(x) = \frac{X_*(x)}{\pi}\int\limits_{-l_0}^{l_0}\frac{q(t)+f(t)}{X_*(t)(\sin(\pi t/\omega)-\sin(\pi x/\omega))}\,\frac{\pi}{\omega}\cos\frac{\pi t}{\omega}\,dt.$$

The algebraization of Eq. (20) is implemented in the same way as in the case of one prefracture zone. As a result, instead of (20) we obtain a nonlinear algebraic system M of equations for approximate values of $q(t_m)$ $(m = 1, 2, \dots, M)$. The numerical solution of this system of equations is constructed as was described above for the case of one prefracture zone.

Using the solution obtained, the expression for the opening of the faces of the prefracture zone can be written as

$$v^+(x,l,\sigma_0) - v^-(x,l,\sigma_0) = \frac{1+\varkappa_0}{2\mu}\int\limits_{0}^{l_0}\frac{[q(t)+f(t)]F_1(t,x)}{X_*(t)}\,\frac{\pi}{\omega}\cos\frac{\pi t}{\omega}\,dt,$$

where

$$F_1(t,x) = X_*(x) + \frac{1}{2}\,X_*(x)\ln\frac{X_*(t)-X_*(x)}{X_*(t)+X_*(x)}.$$

The critical external loads at which cracks form are given by the relations

$$\frac{1+\varkappa_0}{2\mu}\int\limits_{0}^{l_0}\frac{[q(t)+f(t)]F_1(t,x_0)}{X_*(t)}\,\frac{\pi}{\omega}\cos\frac{\pi t}{\omega}\,dt = \delta_c.$$

In addition, it is possible to use condition (17).

Analysis of the initiation of a crack-type defect in a stiffened plate under loading reduces to a joint parametric study of the resolving algebraic system of the problem and the crack formation criterion (17) for various values of the free parameters of the stiffened plate (the mechanical characteristics of the plate and stringer materials and the geometrical dimensions of the stiffening members).

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